

MATH EXPRESSIONS*

MULTIPLICATION & DIVISION



"Basic Facts"

Strategies

- Count by n for multiplication and division
- Square numbers (the product of a whole # and itself)
(ex. $4 \times 4 = 16$, 16 is a square #)
- commutativity
 $8 \times 4 = 4 \times 8$
- Inverse operations—think to unmultiply or to find a missing factor
 $8 \times 4 = 32$ so
 $32 \div 4 = 8$
 $32 \div 8 = 4$

Derived Facts—using facts you know to figure out those you don't

- Start with a known, count by n and then add on
 $4 \times 6 = 4 \times 5$ plus 4
- Start with a known fact and double
 $4 \times 8 = 4 \times 4$ then double
- Combine 2 known facts
 $4 \times 6 = 4 \times 4 + 4 \times 2$

Basic multiplication & division facts are extended to larger numbers:

$$\begin{array}{l} 4 \times 8 = 32 \\ 4 \times 8 \times 10 = 320 \text{ so} \\ 40 \times 8 = 320 \\ 4 \times 80 = 320 \\ 4 \times 8 \times 100 = 3200 \\ 40 \times 80 = 3200 \text{ so} \\ 4 \times 800 = 3200 \\ 400 \times 8 = 3200 \end{array}$$

Multiplication problems can be shown as 4×3 or $4 \bullet 3$ or $4 * 3$.

Division problems can be shown as $50 \div 5$ or $\frac{50}{5}$ or $50/5$ or $5 \overline{)50}$

Multiplication & Division Situations: Problem Types

Equal Groups—unknown total
I have 3 bowls. There are 2 oranges in each bowl.
How many oranges in all?

Equal Groups—unknown number of groups (measurement division)
(think: breaking off a chunk or repeated subtraction)
I have 6 oranges. I give 2 to each person who comes in. How many people get oranges?

Equal Groups—unknown group size (fraction division)
(think: fair share or dealing)
I have 6 oranges. I share them equally between Tom, Sue, and Maria. How many oranges does each get?

Array
(think: things in natural rows & columns)
In my album I have 3 rows of pictures with 2 pictures in each row. How many pictures?



Area
(think: clear grid overlaying a region)

Our family owns a piece of land that is 3 miles wide and 2 miles long. How much land do we own?

Combinations
(think: total # of possible outcomes)
For your ice cream sundae you may choose 1 of 3 ice cream flavors and 1 of 2 toppings. How many different sundaes could you make?

Comparison
(think: a comparison using multiplication)
I have a dog who is 2 feet tall. My big brother is three times as tall as my dog. How tall is he?

Algorithms

Multiplication of Whole Numbers:

Rectangle Sections/
Partial Products

$$\begin{array}{r} 23 \times 36 = 828 \\ \begin{array}{|c|c|} \hline 20 & 3 \\ \hline \end{array} \begin{array}{|c|c|} \hline 30 & 6 \\ \hline \end{array} \\ \begin{array}{|c|c|} \hline 20 \times 30 = 600 & 20 \times 6 = 120 \\ \hline 3 \times 30 = 90 & 3 \times 6 = 18 \\ \hline \end{array} \\ \begin{array}{r} 600 \\ 120 \\ 90 \\ + 18 \\ \hline 828 \end{array} \end{array}$$

Expanded Notation

$$\begin{array}{l} 36 = 30 + 6 \\ \times 23 = 20 + 3 \\ \hline 20 \times 30 = 600 \\ 20 \times 6 = 120 \\ 3 \times 30 = 90 \\ 3 \times 6 = 18 \\ \hline = 828 \end{array}$$

Algebraic Notation

$$\begin{aligned} 23 \times 36 &= (20 + 3) \times (30 + 6) \\ &= 600 + 120 + 90 + 18 \\ &= 828 \end{aligned}$$

Shortcut Notation
(common U.S.)

$$\begin{array}{r} 1 \\ 1 \\ 36 \\ \times 23 \\ \hline 108 \\ 72 \\ \hline 828 \end{array}$$

Division of Whole Numbers:

Partial Quotients
(at least or the big 7)

$$\begin{array}{r} 19R3 \\ 12 \overline{)231} \\ \underline{-120} \\ 111 \\ \underline{-60} \\ 51 \\ \underline{-48} \\ 3 \end{array}$$

Expanded Notation

$$\begin{array}{r} 6 \\ 40 \\ 500 \\ \hline 73,822 \\ - 3,500 \\ \hline 322 \\ \underline{280} \\ 42 \\ \underline{42} \\ 0 \end{array}$$

Digit by Digit

$$\begin{array}{r} 546 \\ 7 \overline{)3,822} \\ \underline{-35} \\ 322 \\ \underline{28} \\ 42 \\ \underline{-42} \\ 0 \end{array}$$

Vocabulary Review

"Product" is the answer to a multiplication problem.

"Quotient" is the answer to a division problem.

MATH EXPRESSIONS*

ADDITION & SUBTRACTION

"Basic Facts"

General Strategies

- Counting on...
- Make a 10
 $8 + 5 = 8 + 2 + 3 = 13$
- Partners & Switch Partners
 $6 = 5 + 1 = 4 + 2 = 3 + 3$ etc.
 $6 = 5 + 1 = 1 + 5$

Specific Strategies

- Doubles
 $4 + 4 = 8$
- Doubles +/- 1
 $7 + 6 = 6 + 6 + 1 = 13$
 $7 + 6 = 7 + 7 - 1 = 13$
- Teens as 10 plus n
 $13 = 10 + 3$

Basic Addition & Subtraction Facts are Extended to Larger Numbers:

$$6 + 7 = 13$$

$$60 + 70 = 130$$

$$600 + 700 = 1300$$

Addition & Subtraction Situations: Problem Types

Change Plus/Join

Chris has 6 books on animals. Her parents give her 7 more animal books. How many does she have in total?

Change Minus/Separate

Mike has 13 tickets to the zoo and he gives 6 of them to his cousins. How many does he have left?

Comparison

Carlos has 7 beautiful sea shells. Lee has 13 beautiful shells. How many more does Lee have than Carlos?

Collection: Part-Part-Whole

In her bedroom, Lynn has a shelf full of stuffed animals. Six are red and 7 are purple. How many does she have in all?

More information is available on the district website:

<http://www.district279.org/departments/curriculum/Subjects/Math/resources.cfm>

Additional NCTM Websites

<http://figurethis.org/>
<http://illuminations.nctm.org/>
 (activities, lessons, games)



Algorithms

Addition of Whole Numbers

Show All Totals/Partial Sums

$$\begin{array}{r} 237 \\ + 59 \\ \hline 200 \\ 80 \\ + 16 \\ \hline 296 \end{array}$$

New Groups Below

$$\begin{array}{r} 237 \\ + 59 \\ \hline 296 \end{array}$$

New Groups Above
(common U.S.)

$$\begin{array}{r} 1 \\ 237 \\ + 59 \\ \hline 296 \end{array}$$

Subtraction of Whole Numbers

Expanded Method

$$\begin{array}{r} 120 \\ - 130 - 16 \\ 136 = 100 + 30 + 6 \\ - 47 = - 40 - 7 \\ \hline 80 + 9 = 89 \end{array}$$

Ungroup First Then
Subtract Everywhere Method/
Trades First

$$\begin{array}{r} 12 \\ 0 \text{ } \cancel{1} 3 \text{ } 16 \\ - 47 \\ \hline 89 \end{array}$$

left to right or right to left

Alternating Ungroup &
Subtract Method
(common U.S.)

$$\begin{array}{r} 2 \\ 1 \text{ } \cancel{3} \text{ } 16 \\ - 47 \\ \hline 9 \end{array}$$

step one

$$\begin{array}{r} 12 \\ \cancel{1} \text{ } \cancel{3} \text{ } 16 \\ - 47 \\ \hline 89 \end{array}$$

step two

= Sign Review

= can also be read as "the same value as" or "is" or "is the same as"
 "The equals sign does not mean "the answer comes next."

Vocabulary Review

"Sum" is the answer to an addition problem.

"Difference" is the answer to a subtraction problem.

*Developed from Math Expressions (Houghton Mifflin Harcourt, 2009) *Based on the work of Edmonds School District #15, Lynnwood, WA